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Approach of Poiseuille's law with the help of sensor and computer technology

I.A. Sianoudis[1], E. Drakaki[2]

1. Department of Physics, Chemistry & Materials Technology, Technological Educational Institute, (TEI) of Athens, Ag. Spyridonos, 12210 Egaleo, Greece, e-mail: jansian@teiath.gr
2. Physics Department, National Technical University of Athens, Zografou Campus, 15780, Athens, Greece, email: drakaki@central.ntua.gr,

Abstract
Simple interferences based on the use of computer and sensor technology, are often proposed and applied in known educational experiments, with successful results, giving them a modern renewal with a new educational profit. In framework of this approach, the present study proposes a different experimental set up for the verification and the confirmation of Poiseuille-Hagen’ law, concerning the flow of real fluids through tubes, with considerable and useful applications in technology and medicine.
In the proposed educational procedure experimental measurements of fluid outflow are performed, via motion sensor and using a computer program, for the determination of hydrostatic pressure and the flow rate respectively. The dependence of the flow rate by parameters as viscosity of the fluid, length and radius of the tube and the pressure difference between the ends of the tube, are objects that allows an educational useful activity in the laboratory for first year students of technological faculties.

1. Introduction

There is a considerable interest in the research of the flow of real fluids in multiple scientific and technological applications. Universities educational studies are requested to help the students to be familiarized with the theory of the laws regulating flow. The knowledge of these laws is very important, especially in hemorheology and hemodynamics, both fields of physiology, in physical and chemical investigation and in the developments of industrial engineering projects [1- 4].
Gotthilf Heinrich Ludwig Hagen and Jean Louis Marie Poiseuille studied and formulated the Hagen-Poiseuille's law, a physical law concerning the voluminal laminar stationary flow Φ of incompressible uniform viscous liquid through a cylindrical tube with the constant circular cross-section.
The important feature of both of these results is the sensitive dependence upon either the channel width or the pipe radius. For instance, for a pipe with a fixed pressure gradient, a 20% reduction in the pipe radius leads to a 60% reduction of the flow rate!
This clearly has important implications e.g in physiological sciences -- small amounts of plaque accumulation in arteries can lead to very large reductions in the rate of blood flow [5].
In general changes regarding the pressure difference between the ends of the tube (ΔP), the tube radius Ro and length L, and the circulating fluid, represented by the viscosity η, can influence the volume flow rate in more or less degree [2].

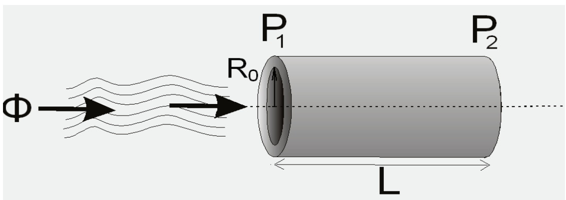

Fig 1: Voluminal laminar flow Φ of uniform viscous liquid through a cylindrical tube with the constant circular cross-section.

The equation of Poiseuille–Hagen’s law is expressed by the equation [6-8]:

$$\Phi = \frac{dV}{dt} = u\pi R_o^2 = \frac{\pi R_o^4}{8n}\left(-\frac{\Delta P}{\Delta x}\right) = \frac{\pi R_o^4}{8n}\frac{\Delta P}{L}$$

$$\Rightarrow \Phi = -\frac{\pi R_o^4}{8n}\frac{\rho g}{L}h(t) \Rightarrow \qquad [1]$$

$$\Phi = -\frac{\pi R_o^4}{8n}\frac{\rho g}{L}h_o \exp\left[-\frac{\pi R_o^4}{A8n}\frac{\rho g}{L}\cdot t\right]$$

where *Φ* is the flow rate, equal to the fluid volume per unit time, which outflows through the pipe, h_O is the initial height of the fluid at the container, while h(t) present the height at each time moment, A is the surface opening of the container, *ρ* and *η* the density and viscosity of the fluid, *P the* pressure. In addition

R_o and L are the radius and the length of the pipe respectively.

2. Materials and Methods

These experiments have been designed so that students are able to verify directly the laws that govern the flow of fluids and how they critically depend on parameters such as the pressure, the length or the diameter of the tubes. The experimental set-up needed to infer Poiseuille's law must allow variation of all the magnitudes that intervene in fluid flow through tubes (ΔP, R_o, L and η).

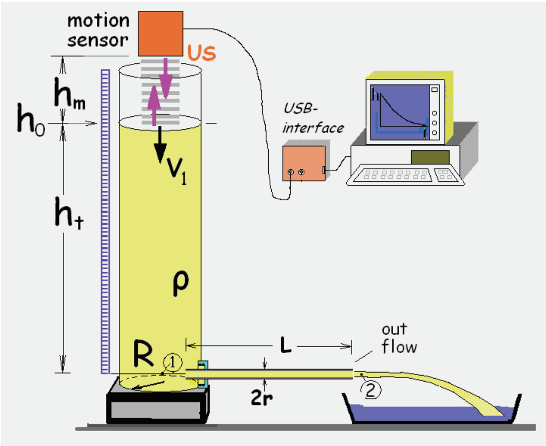

Fig. 2: A schematically draw of the experimental set up used for the measurements

<u>The set-up proposed:</u>
A cylindrical container with the study fluid, a tubes of different radius (0,2-2,5 mm) and lengths (5-70 cm), the liquids of various viscosity and the digital sensor of motion with the appropriate PC interface and Software, e.g. (PS-2103 with the PS-2100 and DataStudio of Pasco).

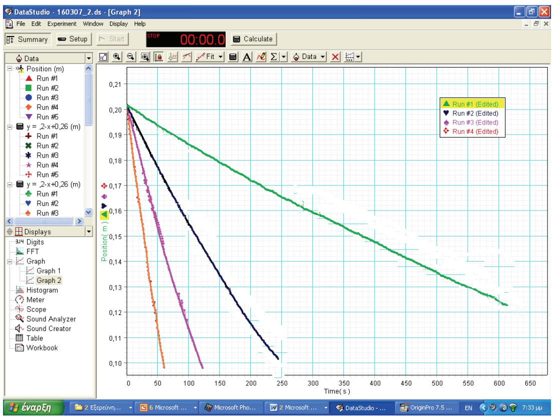

Fig 3a: The print screen of the DataStudio with curves of the heigth vs time as measured by variation the tubes radius

This set up is based on an older version of them, by which the flow rate was measured continues and indirectly via a force sensor by weighting of the out flowed liquid [8].

<u>The measurement method is as follows:</u>
In our experiment we have used tubes of various length and radii. They were common laboratory pipettes of different volumes.
The fluid (de-ionized. Water, Glycerin or their mixture) has been placed in the cylindrical container, filled until a certain, inirtial height H_O (Fig. 2). As soon as we allow the fluid to flow out through the tube the motion sensor at the top starts via the computer program to monitor the difference of the fluid height (the free level fall) into the container. After a settled period of time, by stopping of the measurement, the result appears in form of raw data for further process as seen in fig. 3a. According to the experimental scenarios, the tubes were exchanged with tubes different in Length or in radius.

The data processing provides an appropriate fitting to the experimental data, in order to obtain the right function, which describes the physical phenomenon. The measured data h(t)=f(t) can be converted into the following functions Φ=f(t) and Φ=f(ΔP), according the Poiseuille-Hagen's law, where Φ is the deduced flow rate and P the pressure. A sequence of other graphics can be obtained, as shown in figs. 5-8

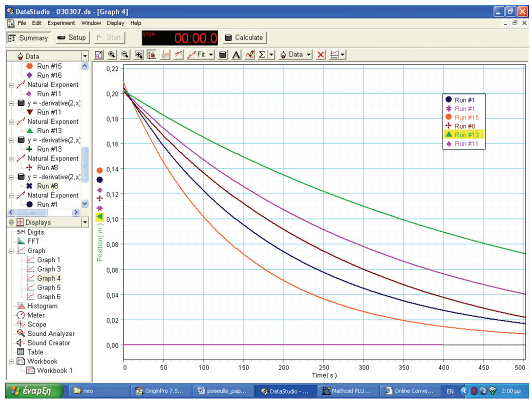

Fig. 3b: The print screen with fitted curves of the heigth vs time for finding the function of the derivates and the flow rates.

3. Results and Discussion

<u>a) Different pressure:</u> Results, according the flow rate Φ, of the liquid versus pressure ΔP, can, easily and straightforward, be derived, as far as the decrease of the height level of the liquid is a continuous process. From the measured data hydrostatics pressure of the liquid (head) can be extracted.
From the graph (fig. 4), the linear mathematical relation between flow rate (Φ) and pressure (ΔP) for different lengths (L1 – L5, 20 – 65 cm) of the tubes under operation, is in accordance with the Hagen-Poiseuille's law.

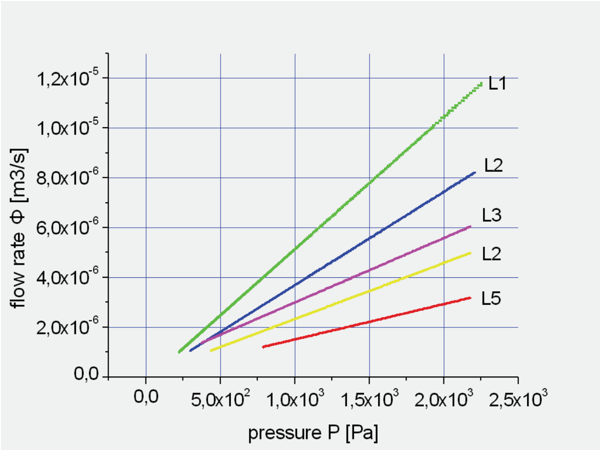

Fig. 4: Flow rate Φ as a function of the pressure P for tubes of $R = 0.88$ mm and different lengths, L.

b) Different Tube lengths: In the figure 5 flow rate versus different length of the tubes under operation is shown. The fitting procedure almost confirmed the mathematical linear relation of $\Phi \approx k\cdot(1/L)$, where k a constant factor [1, 2, 6, 7].

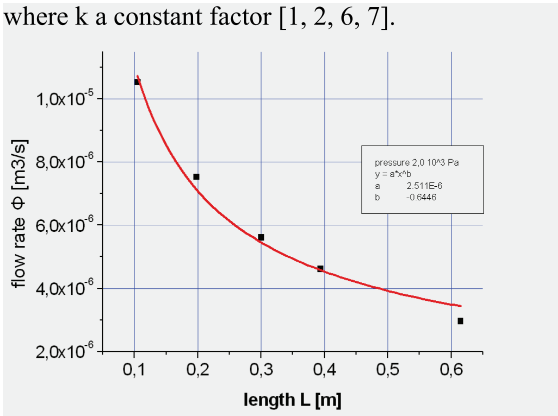

Fig. 5: Flow rate Φ as a function of the length of tubes of $R = 0.88$ mm.

c) Different Tube radius: At the figures 6 and 7 flow rate versus the different radius of the tubes under operation is shown (Length, pressure, and viscosity are kept constant) [1, 2, 6, 7].

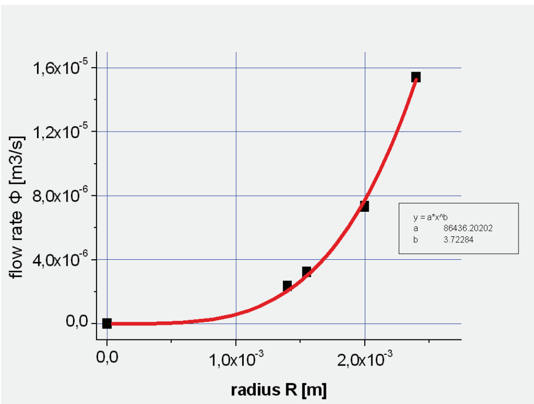

Fig. 6: Plot of the flow rate, Φ, as a function of the radius, for tubes of equal length (0.30 m) and of the same solution.

The mathematical relation of the dependence of the flow rate with the forth power of the radius of the tube could be deduced

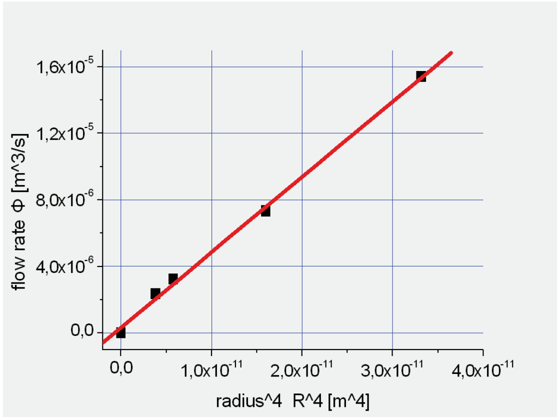

Fig. 7: Plot of the flow rate, Φ, as a function of the forth power of radius, for tubes of equal length (0.30 m) and of the same solution.

4. Conclusions

With the proposed exercise, the student is able to infer Poiseuille's law, which determines the flow rate of a fluid through a tube by means of an experimental design that clearly illustrates the different steps of the scientific method.

The proposed set-up is relatively simple and inexpensive, and offers very satisfactory results. These laws are behind many physiological phenomena of the circulatory system, either normal or pathological, such as the local regulation of bloodflow, the narrowing of a vessel, the generation of a critical stenosis, etc. Even though precise measurements of the relationship between pressure and flow rate can be done with the proposed experimental setup, its construction remains quite simple and the cost is reasonable.

5. References

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This work is represented in form of poster at the "Frühjahrstagung der Deutschen Physikalischen Gesellschaft des Fachverbandes Didaktik der Physik" in Regensburg 2007.

Photos: a) and **b)** of the experimental set up, **c)** of the pipettes used as tubes with certain radius and **d)** of the outflow.